International Journal of Engineering, Science and Mathematics
Vol. 7 Special Issue 4(2), April 2018,
ISSN: 2320-0294 Impact Factor: 6.765
Journal Homepage: http://www.ijmra.us, Email: editorijmie@gmail.com
Double-Blind Peer Reviewed Refereed Open Access International Journal - Included in the International Serial Directories Indexed & Listed at: Ulrich's Periodicals Directory ©, U.S.A., Open J-Gage as well as in Cabell's Directories of Publishing Opportunities, U.S.A

On Graphs of Alternating Knots

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Abstract

A knot diagram is a picture of a projection of a knot onto a plane. Usually, only double points are allowed (no more than two points are allowed to be superposed), and the double or crossing points must be "genuine crossings" which transverse in the plane. The aim of this paper is to investigate some graphical properties of alternating knots(links) with the help of generalized form of Reidemeister moves and flype moves. It has been shown the equivalency of the associate graphs diagrammatically.

Keywords:

associate graphs;
d-signed graphs;
Reidemeister moves;

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1. Introduction

A projection of a knot or a link on a two-dimensional plane divides the plane into several regions. It is a useful method to separate these regions into two classes, unshaded(called white regions) and shaded(called black regions) in the study of knot theory.To simplify the above method, C.Bankwitz, introduced the notion of graph of knot in his study of alternating knots. A connection between knot theory and graph theory has firstly been established by Reidemeister [3]. In this paper we consider the graphs of knots(links) as described in [1]. Here we extend our previous results with the establishment of the equivalence between $G(\pi)$ and $G'(\pi)$. To do this we consider $G(\tilde{R})$ move, a generalize form of the Reidemeister [3] moves of type II and III. In [1] we have found some relative properties between the knot theory and the graph theory via graph theoretic approach. In the present paper we have shown a diagrammatic relationship between the graphs corresponding to the white(black) and black(white) regions of the same alternating knot(link) diagram. In this process we consider the planar graphs from the graph theory.

2. Preliminaries

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The definitions of projection of knot, alternating knots(links), amphicheiral knots and *d*-signed graphs are given in [1]. A topological graph *G* is said to be *k*-valent if the number of arcs incident with each vertex is equal to *k*. Let π be a regular normed projection of a knot on a two dimensional sphere [1]. Let $G(\pi)$ be the graph corresponding to white(black) regions of a given knot then by the associate graph of $G(\pi)$ we mean the graph corresponding to the black(white) regions of the same knot and it will be denoted by $G'(\pi)$.

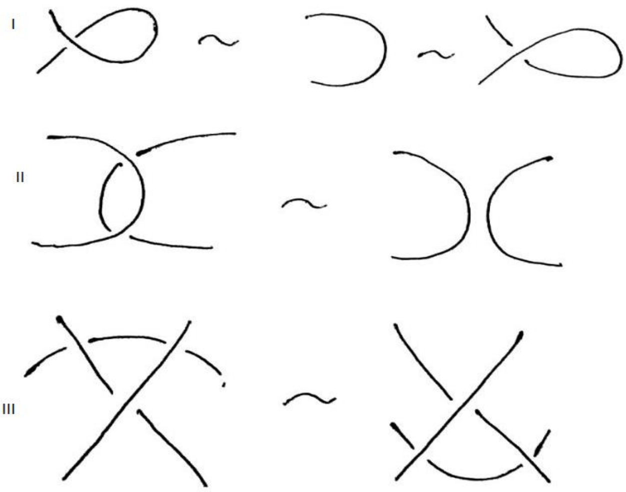
Now recall that

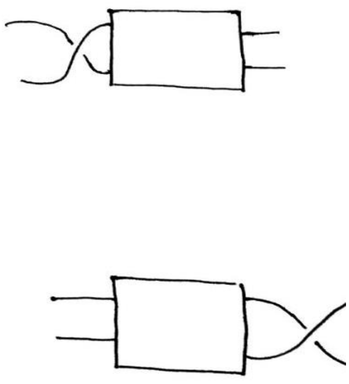
Remark 2.1: Thus for a given regular projection π of a knot, we have two graphs $G(\pi)$ and $G'(\pi)$ i.e., every knot has a pair of graphs, associate to each other. In reverse order we have, for a given graph, whichever $G(\pi)$ or $G'(\pi)$, there is a uniquely determined regular projection π.

Remark 2.2: Let π and π' be regular projection of knots D and D´respectively, which are mirror images to each other. Then $G(\pi)$ and $G(\pi')$ are of the same type and have the opposite signs. $G(\pi')$, the conjugate graph of $G(\pi)$.

We consider a diagrammatic relationship between knots and planar graphs. To do this we consider the Reidemeister moves[3] and the generalize Reidemeister moves with *flype* [3] moves, a flype is a move on a tangle(with two inputs and two outputs) obtained by rotating the tangle 180 degree .

All are given below:





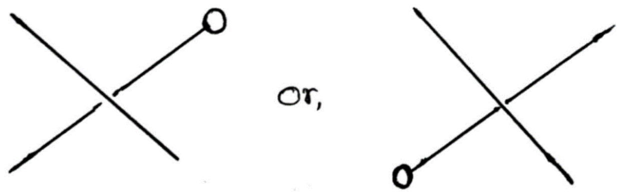
Reidemeister moves Flype moves

3. Results and Analysis

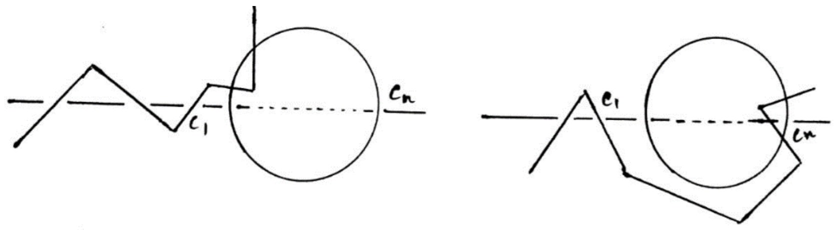
Now we consider the two propositions.

Proposition 3.1: In an alternating knot(link) diagram, $G(\tilde{R})$ move is well defined with respect to Reidemeister moves.

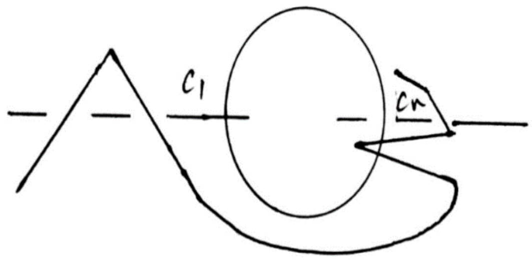
Proof: Suppose the knot (link) diagram has *n* number of crossings in the part of.



Let c_1 be the first crossing countered while going from left to right and c_n be the last crossing.
Now we performing the Reidemeister moves (type II) twice in the diagram.



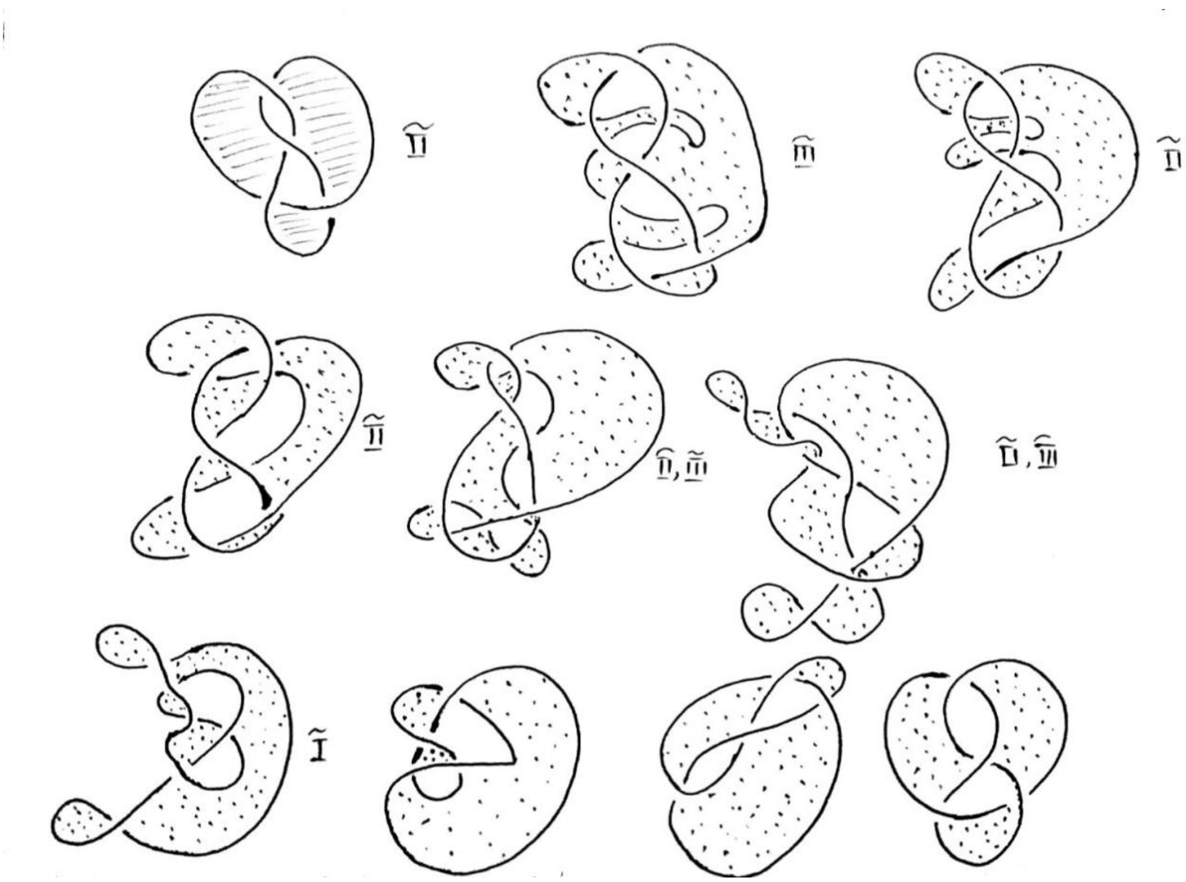
Now, at the crossing c_1 we perform the Reidemeister moves of type III. We perform it and continue performing a finite sequence of Reidemeister moves of type I, II, III as per requirement on all the crossings in between the crossings c_1 and c_n.
Here, at the crossing c_n we perform the Reidemeister move of type III and lastly we get the following.



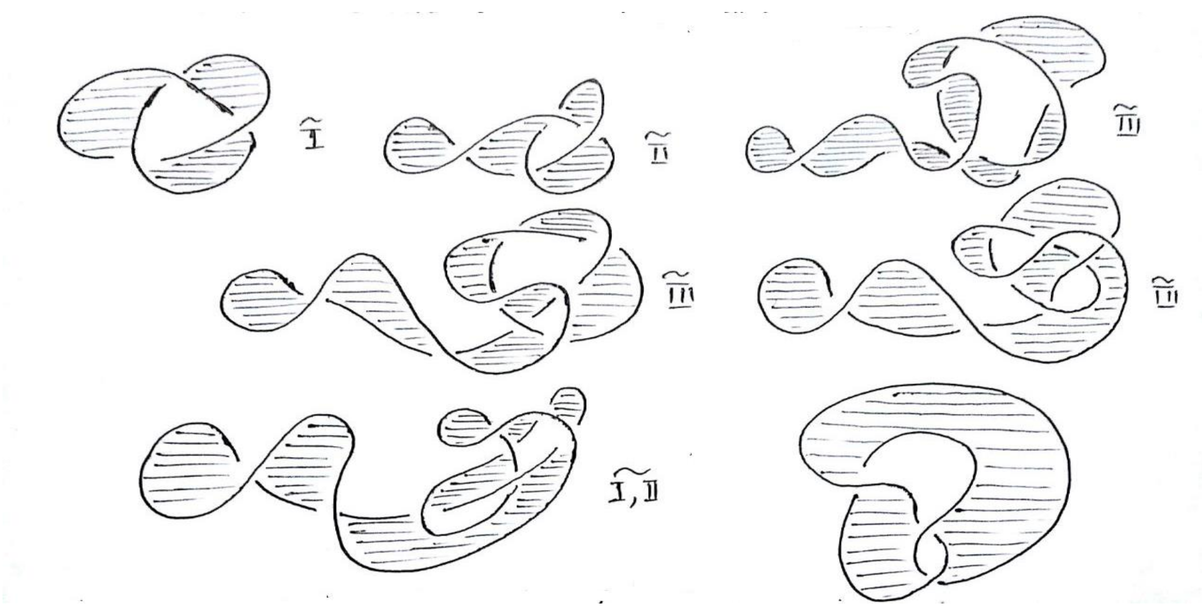
Proposition 3.2: In an alternating knot(link) diagram, *flype* and a *twice−flype* moves are both well defined by Reidemeister moves.

Proof: It follows from the $G(\tilde{R})$ move.

Now we see the effects of $G(\tilde{R})$ move on two alternating knots (both types of amphicheiral and non-ampicheiral).



Amphicheiral knot (or figure-eight knot)



Non-amphicheiral knot (or clover-leaf knot)

From the above results and discussion we observe that $G(\tilde{R})$ move changes the white regions into black regions and vice versa. Therefore, the *d*-signed graph corresponding to the white(black) regions changed step by step to the *d*-signed graph corresponding to the black(white) regions of the same knot(link) diagram. So that the associate graphs [1] can be change into one another by the Reidemeister moves and $G(\tilde{R})$ move. Thus we have the following theorem:

Theorem 3.3: Associate graphs are equivalent by Reidemeister moves.

Sketch of the Proof: We have already established that knots(links) are in one-to-one correspondence with connected planar *d*-signed graphs [1]. The newly generated $G(\tilde{R})$ move changes the black(white) regions into white(black) regions and vice-versa. By this way we get that the associate graphs $G(\pi)$ and $G'(\pi)$ are equivalent to one another. By this way we get a new way to describe the mirror image specially when an unbounded shaded(unshaded) region is change into a bounded unshaded(shaded) region and the labelling also changed step by step and accordingly without changing any over crossing(under crossing) into under crossing(over crossing) i.e., the underlying universe remains unchanged. From the above discussion and results we can have the following corollary:

Corollary 3.4: A reduced alternating knot(link) is amphicheiral if and only if the graph corresponding to unshaded region is isomorphic to the graph obtained via $G(\tilde{R})$ move.

4. Conclusion

Thus, we get a new way to define the mirror image via Reidemeister moves (with its generalize move). This will provide us to further study of alternating knots (links).

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